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Design Document for Differential GPS Ground Reference Station Pseudorange Correction Generation Algorithm

Wright Aeronautical Laboratories
Wright-Patterson Air Force Base, OH 45433

December 1986
Interim Report

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16 Abstract

The pseudorange correction generation algorithm takes the satellite measurements and processes them through a filter to remove the receiver-unique error sources, which tend to be higher frequency than locally common error sources. An estimate of clock error is computed and then removed equally from all filter outputs. The clock-corrected filter outputs are coefficients of a quadratic function in time estimating pseudorange error. A best-fit linear approximation to this function over the applicable interval is generated; the coefficients of this linear time function are the elements to be broadcast as a pseudorange correction.

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PREFACE

This Interim Report was prepared by Wright Aeronautical Laboratories at Wright-Patterson Air Force Base in Ohio, on a reimbursable agreement with the U.S. Department of Transportation, Transportation Systems Center (TSC), Cambridge, MA. TAU Corporation of Los Gatos, CA, served as subcontractor for this project. The work was co-sponsored by the U.S. Coast Guard Office of Navigation, Office of Research and Development; and by the Research and Special Programs Administration. J. H. Kraemer of the Center for Navigation at TSC was the Technical Monitor, assisted by R. M. Kalafus.

The algorithms described in this report determine the differential corrections to be broadcast to users of the Global Positioning System (GPS) who require higher accuracy navigation or position information than the 30 to 100 meters that GPS normally provides. Properly equipped marine users receiving the broadcasted corrections are expected to achieve better than 10 meters accuracy with the Differential GPS system.

The Differential GPS program involves the fabrication of a differential GPS testbed, consisting of an 8-channel GPS receiver for the reference station, data processing hardware and software to generate the corrections and perform channel management and integrity functions, user receiver, and data acquistion hardware and software. The system will be tested in 1987.

Other reports which are or will soon be available include a specification of the design, a description of the integrity management function algorithms, and test results.

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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in²	square inches	6.5	square centimeters	cm²
ft²	square feet	0.09	square meters	m²
yd²	square yards	0.8	square meters	m²
mi²	square miles	2.6	square kilometers	km²
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft³	cubic feet	0.03	cubic meters	m³
yd³	cubic yards	0.76	cubic meters	m³
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 cm (exactly). For other exact conversions and more detail tables see NBS Misc. Publ. 286. Units of Weight and Measures. Price $2.25 SD Catalog No. C13 10 286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm²	square centimeters	0.16	square inches	in²
m²	square meters	1.2	square yards	yd²
km²	square kilometers	0.4	square miles	mi²
ha	hectares (10,000 m²)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m³	cubic meters	36	cubic feet	ft³
m³	cubic meters	1.3	cubic yards	yd³
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

°F   -40   0   32   98.6   212
°C   -40   -20   0   20   37   40   60   80   100

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1.0 SYSTEM CONCEPT

The differential GPS (DGPS) reference station is a GPS receiver system fixed to a known, surveyed location, measuring GPS ranges to all satellites in view, and computing and broadcasting range bias error corrections to local users. The user can then remove the locally common bias errors from his own measurements before computing his position/time coordinates. The resulting DGPS navigation solution is substantially more accurate than a simple stand-alone GPS navigation solution with only minor changes to the user equipment.

2.0 ALGORITHM REQUIREMENTS

The pseudorange corrections generated by the Pseudorange Correction Generator (PRCG) must accurately estimate the bias errors common to local users. To do this, the reference receiver computes nominal range to the satellites and compares against the measured range. Error sources that are not locally common, i.e., receiver-unique error sources, must be removed from the measurements before broadcasting. Table 1 contains an example list of error sources, both receiver-unique and locally common, with their amplitudes, frequencies, and correlation distances. Even though the parameters shown in Table 1 are only rough order of magnitude, the table does convey the relative comparisons common to reference receivers and supports the rationale behind the PRCG algorithm.

Table 1. GPS Error Sources

Error Source	Amplitude	Correlation Time	Correlation Distance
Receiver Noise (carrier/code)	.01/10 m	<10 sec	0.
Interchannel Bias	1 m	500 sec	0.
Multipath	20 m	500 sec	10 m
Tropospheric Delay	30 m	1000 sec	20 km
Satellite Error (SA)	100 m	100 sec	400 km
Ionospheric Delay	60 m	1000 sec	1000 km
Satellite Error (no SA)	5 m	10000 sec	1000 km

The last four error sources, tropospheric and ionospheric delay, and satellite error, are exactly the locally common bias errors which DGPS is designed to eliminate. (Satellite error includes position/time errors resulting from incorrect satellite orbit and clock parameters in Space Vehicle (SV) Data Blocks I and II.) The other three error sources must be removed before the corrections can be computed.

The first, receiver noise, can be effectively removed by filtering. In fact, the heart of the PRCG algorithm is a set of parallel Kalman (linear optimal) filters, one filter for each satellite in view.

Interchannel bias is a problem unique to multichannel receivers such as the target receiver, the Trimble 4000A. The magnitude of this error is relatively small and is easily measured if a spare channel is available. The

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spare channel, or rover, can measure itself against each of the other channels by locking in on the same satellite and comparing measurements over a period of time. The average difference between measurements is the bias relative to the roving channel. The Trimble 4000A performs this procedure among all four channels simultaneously upon initialization and can also be commanded to recalibrate the interchannel biases from the software interface. TAU has studied interchannel bias drift in the 4000A and has determined that it is negligible over a six-hour period. For the demonstration program, no inter-channel bias functions will be implemented other than recalibration at begin-ning and end of the test periods.

The .third error source, multipath, is due to interference from reflected GPS signals. Multipath effects must be removed since they are generally not common to the reference station and user. However, they can be easily mistaken for ionospheric delay, so they must be removed by methods other than filtering. The most effective method is to shield the antenna from all reflective sources, i.e., build an antenna mask that allows only a clear-sky view. For a stationary antenna such as a DGPS reference station, this is tractable. Another method of removing multipath effects is to estimate the multipath error and compensate, using a large data base of previous days' readings. This may be possible because multipath patterns should be identical from day to day, assuming the satellite rises/sets in the same direction and the multipath sources (buildings, lakes, etc.) do not change. The third method of eliminating multipath is a multiple antenna approach, assigning a satellite to the antenna least susceptible to multipath for that satellite at that time. The current demonstraton program assumes multipath has been removed by the first method, as it is the most likely to be implemented in operational systems.

Upon elimination of noise and multipath effects, only locally common errors are left. Among these, troposphere may decorrelate over relatively short distances, so special provision is made in the RTCM format for distant users to recompute a tropospheric correction. Ionosphere and satellite orbit errors are two of the major drivers of the differential concept, because they are correlated over large distances, not as easily modeled as troposphere, and large enough to preclude precision navigation.

Table 1 reveals a major requirement for proximity in a temporal sense. Most error sources are relatively stable over time, with the exception of severe Selective Availability conditions and possibly rare tropospheric or ionospheric disturbances. These cause quick, unpredictable changes in the bias errors and motivate using update rates that are higher than normal circumstances would dictate. Without these unstable error sources, error dynamics are fairly small over a few minutes; but in the presence of these error sources, the accuracy of the corrections may degrade over seconds. The solution to this problem requires a balance between expected rate of correction degradation, the accuracy requirements, broadcast message content, and available bandwidth. The RTCM format compromises on a nominal 50-bit per second data rate with a 300-bit message (nominal eight satellites in view), yielding a six-second update rate. Not only are the corrections broadcast, but also the rate of change of the corrections, so that temporal degradation is confined to the second-order effects over a few seconds. Figure 1 shows correction accuracy degradation for this scheme over time, assuming Selective Availability behavior matches the limited data sets currently available to the

public [1]. A six-second update rate corresponds to accuracy on the order of five meters, sufficient for this demonstration project. Six seconds also corresponds neatly to current GPS broadcast formats: the RTCM message boundaries coincide with the Z-count. This provides a convenient time reference for the corrections as well; the broadcast correction is the estimated pseudorange correction for the particular satellite at the beginning boundary of the message, allowing the user to easily extrapolate it to the current time using the correction rate of change that is also broadcast.
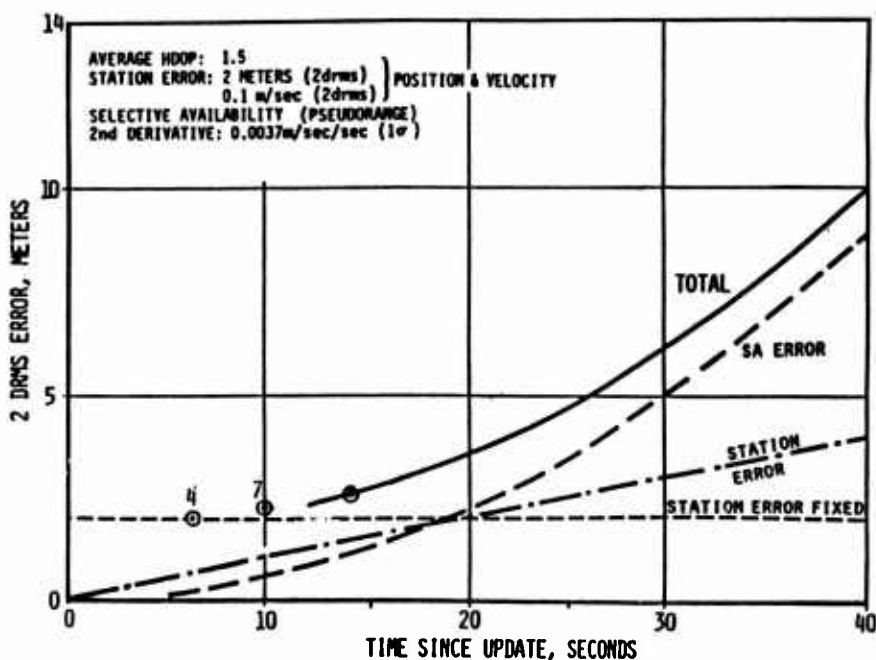

Figure 1. Differential GPS Error Growth Due to Selective Availability

3.0 ALGORITHM DESCRIPTION

Each satellite in view has a dedicated Kalman filter/correction generator. As illustrated in Figure 2, the only co-mingling of satellite data occurs in the clock correction filter, which is a heavily-damped average of all pseudorange measurements. The clock correction filter is described in more detail after the following sections describing the individual Kalman filters.

The Pseudorange Correction Generator accepts as inputs pseudorange and deltarange measurements, satellite ephemeris data and clock correction estimates; and outputs pseudorange error estimates (to the clock filter), pseudorange correction and correction rates of change, and difference between old and new ephemeris. Residuals from the filter are stored also and used for integrity management purposes.

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Figure 2. Pseudorange Correction Generation

3.1 Measurement Processing

The raw measurements from the receiver are pseudorange and deltarange; before input to the filter, they must be transformed to pseudorange and delta-range error by subtracting the nominal range and change in range as well as the SV clock corrections. The data to perform these computations come from SV Data Blocks I and II; the general data flow is illustrated in Figure 3. The formulas for clock correction, satellite position, and range are in Tables 2, 3, and 4, respectively.

Nominal deltaranges are obtained by computing the nominal pseudoranges at the beginning and the end of the averaging time interval, and then backwards differencing.

Figure 3. Pseudorange Computation

Table 2. Satellite Clock Corrector

Variables

tot, tor — time of transmission, reception

SV, RVR, GPS — SV, receiver, GPS time frames (subscripts)

PR_{meas} — measured pseudorange

Δt_{SV} — SV clock error

toc, af_o, af_1, af_2, tgd — SV Data Block I parameters

e, $\sqrt{A}$, sin E — orbital parameters (cf. Table 3)

Equations

$tot_{SV} = tot_{RVR} - PR_{meas}$

$$\Delta t_{SV} = (af_o + tgd) + af_1(tot_{SV} - toc) + af_2(tot_{SV} - toc)^2$$
$$+ (-4.42809305 \times 10^{-10})\, e \sqrt{A} \sin E$$

$tot_{GPS} = tot_{SV} - \Delta t_{SV}$

Table 3.   Satellite Orbit Equations

Variables

$\mu, \dot{\Omega}_e$ — earth gravity, rotation constants (WGS 72)

toe, M_0, A, Δn, e, ω, C_{us}, — SV Data Block II parameters

C_{uc}, C_{rs}, C_{rc}, C_{is}, C_{ic},

i_0, $\frac{di}{dt}$, Ω_0, $\dot{\Omega}$

M, E, v — mean, eccentric, true anomalies

u, r, i, Ω — inertial orbital elements

X_{SV}, Y_{SV}, Z_{SV} — SV position in ECEF at time of transmission

Equations

$$t = tot_{GPS} - toe$$

$$M = M_0 + \left(\sqrt{\frac{\mu}{A^3}} + \Delta n\right)t$$

$$M = E - e \sin E \qquad \text{solved by Newton's root-finding method, precision} <10^{-12} \text{ radians}$$

$$v = \tan^{-1}\left\{ \frac{\sqrt{(1-e^2)} \sin E}{\cos E - e} \right\}$$

$$\phi = v + \omega$$

$$u = \phi + C_{us} \sin 2\phi + C_{uc} \cos 2\phi$$

$$r = A(1-e \cos E) + C_{rs} \sin 2\phi + C_{rc} \cos 2\phi$$

$$i = i_0 + \left(\frac{di}{dt}\right) t + C_{is} \sin 2\phi + C_{ic} \cos 2\phi$$

$$\Omega = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e)t - \dot{\Omega}_e t_{ce}$$

$$x_{SV} = r \cos u \cos \Omega - r \sin u \cos i \sin \Omega$$

$$y_{SV} = r \cos u \sin \Omega + r \sin u \cos i \cos \Omega$$

$$z_{SV} = r \sin u \sin i$$

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Table 4. Nominal Pseudorange Computation

Variables

$\dot{\Omega}_e$, c   earth rotation rate, speed of light

$\overline{SV}$, $\overline{RR}$ ECEF satellite, receiver positions

$\overline{x}_{TX}$, $\overline{x}_{RX}$ vector coordinates in ECEF at time of transmission and reception

ε angle earth rotates between t_{TX}, t_{RX}

r   range between SV and reference receiver

pr_{nom} nominal pseudorange

rto   receiver time offset (dual receivers only)

Equations

$$\varepsilon = |\overline{SV}_{TX} - \overline{RR}_{RX}| \cdot \dot{\Omega}_e/c$$

$$\overline{SV}_{RX} = \begin{bmatrix} \cos\varepsilon & \sin\varepsilon & 0 \\ -\sin\varepsilon & \cos\varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \overline{SV}_{TX}$$

$$r = |\overline{SV}_{RX} - \overline{RR}_{RX}|$$

$$pr_{nom} = r + \Delta t_{SV} - rto$$

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Note that nominal pseudorange as defined here does not include the effects of ionosphere or troposphere, nor does it include the effect of receiver clock error. Therefore, these effects <u>will</u> be included in the pseudorange/deltarange error measurements entering the filter. The clock error alone is on the order of hundreds of kilometers, dwarfing all other pseudorange error sources combined. As a result, the clock error is common to all satellites in view, however, and is estimated and removed after filtering.

3.2 Kalman Filter Processing

Each satellite in view is assigned a dedicated four-state Kalman filter. The filter processes pseudorange and deltarange measurement upon receipt and outputs a pseudorange error function, quadratic in time, suitable for input to the pseudorange correction formulas.

The filter will be defined here according to the Kalman filter equations shown in Table 5 [2]. The actual arithmetic implementation is the U-D formulation laid into a standard software version by G. J. Bierman [3]. Using the terminology of Table 5, the filter is defined as follows:

$$\underline{x}_k = \text{state vector at time } t_k = \begin{bmatrix} \text{PRE} \\ \dot{\text{PRE}} \\ \ddot{\text{PRE}} \\ \dot{\text{I}} \end{bmatrix}$$

where PRE – pseudorange error $\ddot{\text{PRE}}$ – pseudorange error acceleration
       $\dot{\text{PRE}}$ – pseudorange error rate $\dot{\text{I}}$ – ionospheric delay rate

$$\underline{w}_k = \text{state noise } t_{k-1} \text{ to } t_k = \Phi_{k-1} \begin{bmatrix} 0 \\ 0 \\ \text{PR acceleration noise} \\ \text{ionospheric delay rate noise} \end{bmatrix} ;$$

$$\Phi_{k-1} = \begin{matrix} \text{transition matrix} \\ t_{k-1} \text{ to } t_k = \end{matrix} \begin{bmatrix} 1 & (t_k - t_{k-1}) & 1/2(t_k - t_{k-1})^2 & 0 \\ 0 & 1 & (t_k - t_{k-1}) & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ;$$

$$Q_k = \text{state noise covariance} = \Phi_{k-1} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & q_P & 0 \\ 0 & 0 & 0 & q_I \end{bmatrix} \Phi_{k-1}^T$$

with q_P = (TBD) and q_I = (TBD);

$\underline{z}_k$ = (pseudorange error measurement, deltarange error measurement);

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \end{bmatrix} ;$$

$$R_k = \begin{bmatrix} r_{code} & 0 \\ 0 & r_{carrier} \end{bmatrix}$$

with r_{code} = (TBD) and $r_{carrier}$ = (TBD).

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Table 5. Summary of Kalman Filter Equations

System Model	$\underline{x}_k = \Phi_{k-1}\underline{x}_{k-1} + \underline{w}_{k-1}, \quad \underline{w}_k \sim N(\underline{0}, Q_k)$
Measurement Model	$\underline{z}_k = H_k\underline{x}_k + \underline{v}_k, \quad \underline{v}_k \sim N(\underline{0}, R_k)$
State Estimate Extrapolation	$\hat{\underline{x}}_k(-) = \Phi_{k-1}\hat{\underline{x}}_{k-1}(+)$
Error Covariance Extrapolation	$P_k(-) = \Phi_{k-1}P_{k-1}(+)\Phi_{k-1}^T + Q_{k-1}$
State Estimate Update	$\hat{\underline{x}}_k(+) = \hat{\underline{x}}_k(-) + K_k[\underline{z}_k - H_k\hat{\underline{x}}_k(-)]$
Error Covariance Update	$P_k(+) = [I - K_k H_k]P_k(-)$
Kalman Gain Matrix	$K_k = P_k(-) H_k^T[H_k P_k(-)H_k^T + R_k]^{-1}$

The Kalman filter has three unique features worthy of comment. First, even though linear corrections are being broadcast, quadratic corrections are being computed. As will be demonstrated in the section on pseudorange correction computation, this significantly lowers the latency error. This latency error, due in part to delays in transmission and in part to the six-second applicability error, becomes less sensitive to pseudorange acceleration because these effects are estimated and partly compensated.

The second feature is the presence of the ionospheric delay rate term. This term is included to model the deltarange measurement properly. As shown in Reference 4, the delta-range is not truly the change in pseudorange:

$$\Delta PR - \Delta I = DR + \Delta I$$

$$\text{or } \Delta PR - 2\Delta I = DR$$

where ΔPR, ΔI are change in pseudorange and ionospheric delay, respectively, and DR is the so-called deltarange as measured by the carrier loops. Although the error introduced by ignoring ionospheric delay is small, it can build up over time to several meters. Rather than mismodel the system with no ionosphere and compensate with additional process noise, the extra state has been added.

The last feature deals with Kalman filter process noise modeling and the character of the measurements. Typical Kalman filter states are current instantaneous position, velocity, and acceleration; this filter has instantaneous position and average velocity and acceleration over the time interval. The particular formulation of the state is dictated by the deltarange measurement. In the target receiver, the deltarange measurement is derived from the average Doppler over the interval since the last measurement, as opposed to instantaneous Doppler at the time of measurement. Thus, the deltaranges reflect the total change in range over time instead of regularly sampled range rates. For position estimation, the "average" measurement gives slightly better results than the "instantaneous" measurement because of slightly lower process noise. Consequently, the Trimble 4000A performs range tracking slightly better than receivers using an instantaneous range-rate or Doppler.

3.3 Clock Correction Filter

To reduce the dynamic range of the pseudorange corrections, the average of the pseudorange errors is computed and removed before computing the pseudorange corrections. Each pseudorange error is typically the sum of a very large common term (the receiver clock error) and the satellite-unique term (comprising ionospheric and tropospheric delays and satellite clock and orbit error). Because of atmospheric delays, the pseudorange error average tends to be a bit larger than the receiver clock error. The moniker "clock correction" is used because the difference between the two is so small that correction is almost exactly equal to the receiver clock error. Any reference receiver clock error remaining in the pseudorange correction translates into an equal and opposite user receiver clock error when the differential corrections are applied. Indeed, simple theoretical principles dictate that the correction term can be any number, as long as it is subtracted equally from each of the differential corrections; and that the only term affected is user time, not user position. This is due to the nature of differential GPS, that the user is navigating relative to the receiver in position as well as time, so a receiver clock error (or error in the receiver clock correction) results in only a user time error.

In practice, the above is nearly true; in fact, it is reliable for user receivers such as the Trimble 4000A, which make no effort to model the dynamics of the user clock. For more sophisticated receivers, or receivers aided by and depending on cesium clocks, large clock error dynamics introduced by the differential corrections and left unmodeled by the user navigation filter may correlate into the position estimate and contaminate the navigation accuracy. The amount of contamination depends upon how heavily the user filter relies on the low user clock error dynamics.

At issue in the design of the reference station is whether the reference receiver clock error remaining in the differential corrections has high enough unmodeled dynamics to affect the user position solution. The most reliable approach is to keep the actual clock dynamics low with a rubidium frequency standard, as is being done in this project. Once the clock dynamics have been guaranteed to be very low, a pseudorange average will be able to follow the true reference receiver clock error fairly closely yet not contaminate user position error. The average must be sufficiently damped so that impulses caused by rising or setting satellites or uploads cause minimal disturbance. A simple, two state filter is sufficient to create this damped average. The inputs to the filter are the simple averages of the current pseudorange and deltarange errors. A suboptimal constant gain linear filter will be used:

$$\hat{cc}_{i+1} = \hat{cc}_i + \overset{\triangle}{cc}_i \Delta t + k_{11} \overline{PRE}_i$$

$$\overset{\triangle}{cc}_{i+1} = \overset{\triangle}{cc}_i + k_{22} \, PRE_i$$

where

$\hat{cc}_i$ = clock correction estimate at t_i

Δt_i = time since last measurement cycle, $t_{i+1} - t_i$

$\overline{PRE}_i$ = average of pseudorange error estimates at t_i

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$k_{11} = k_{22} = \dfrac{1}{n}$, where $n = $ (TBD).

A "corrected" pseudorange error and error rate are then generated:

$$\hat{PREc}_i = \hat{PRE}_i - \hat{cc}_i$$
$$\dot{\hat{PREc}}_i = \dot{\hat{PRE}}_i - \dot{\hat{cc}}_i$$
$$\ddot{\hat{PREc}}_i = \ddot{\hat{PRE}}_i$$

Acceleration is unaffected because rubidium clock frequency drift is so small.

3.4 Computation of Linear Pseudorange Correction

Each quadratic pseudorange error function must be converted to a linear pseudorange correction function. Typically, there is a lag between the last available data input into the filter (denoted t_i) and the start of the broadcast interval (denoted t_b) in which the correction will be valid, and there sometimes is variation in the length of the time interval (Δt). (See Figure 4.) These are all considered in the following algorithm, which computes the line of best fit (in a maximum error sense) to the quadratic function predicted by the pseudorange error filter.

The predicted quadratic coefficients are output from the filter:

$$PREc(t) = \hat{PREc}_i + \dot{\hat{PREc}}_i(t-t_i) + \ddot{\hat{PREc}}_i(t-t_i)^2/2.$$

The rate of change of pseudorange correction is equal to the slope of the tangent of the PRE curve at the midpoint of the broadcast interval:

$$\dot{PRC} = \dot{\hat{PREc}}_i + \ddot{\hat{PREc}}_i(t_b-t_i+\Delta t/2).$$

Lastly, the value of PRC is the ordinate of the best fit line at time t_b. The best fit line is midway between the above-mentioned tangent and the secant joining the points of the PRE curve at times t_b, $t_b+\Delta t$:

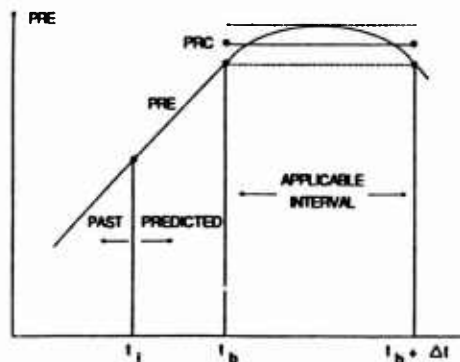$$PRC = PREc(t_b) + \ddot{\hat{PREc}}_i(\Delta t)^2/16.$$

Figure 4. Linear Pseudorange Correction

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4.0 SUMMARY

The above algorithm takes the satellite measurements and processes them through a filter to remove the receiver-unique error sources, which tend to be higher frequency than locally common error sources. An estimate of clock error is computed and then removed equally from all filter outputs. The clock-corrected filter outputs are coefficients of a quadratic function in time estimating pseudorange error. A best-fit linear approximation to this function over the applicable interval is generated; the coefficients of this linear time function are the elements to be broadcast as a pseudorange correction.

REFERENCES

1. Kalafus, R. M., A. J. Van Dierendonck, T. A. Stansell, Jr., and N. A. Pealer, "Recommendations of Special Committee 104 Differential Navstar/GPS Service," Radio Technical Commission for Maritime Services, Washington, DC, November 20, 1985.

2. Gelb, A., id., _Applied Optimal Estimation_, M.I.T. Press, 1974.

3. Bierman, G. J., _Factorization Methods for Discrete Sequential Estimation_, Academic Press, 1977.

4. Jorgensen, P., "Ionospheric Measurements from Navstar Satellites," SAMSO-TR-79-29, 1978.

135 copies